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## Variation Across Districts in Intended Topic Coverage: Mathematics

### Introduction

Studies that examine American education – what is wrong with it and how to fix it – have proliferated over the last half-century. Some studies examine education from all perspectives: home, community, state, regional, district, local, teacher, student – and at several levels within each. Other studies examine education by dissecting a particular approach or concept. One concept, opportunity to learn (OTL), was first introduced in the early 1960s, and continues to be an important consideration in education research. OTL was first defined by Carroll as the coverage of particular content over a given amount of time (Carroll, 1963), and further discussed in Husen’s report for the First International Mathematics Study (FIMS) (Husen, 1967a, 1967b; Floden, 2002). International studies particularly have examined the relationship between achievement results and OTL (Floden, 2002).

Another approach developed in international comparative studies, particularly by the International Association for the Evaluation of Educational Achievement (IEA) (Floden, 2002), is that of thinking about curriculum in terms of three facets: intended, implemented, and attained. The *intended* curriculum is specified by state or district level standards/frameworks, or grade level learning expectations, and is the focus of this paper. Kher (2009) discusses in depth the *implemented* curriculum, the content that teachers cover in the classroom, and the amount of time they devote to the content covered. *Attainment* is measured through the scoring of assessments that are administered to students to examine content knowledge. A previous issue of the PROM/SE Research Report (2006) thoroughly discussed attainment of content knowledge about fractions by elementary and middle grade students.

The practice of keeping decisions about what is taught in our schools at the local level is a tenet that has held its place even with the signing into law of the No Child Left Behind Act (NCLB) of 2001. This paper explores the extent to which implementing curriculum at the local level has created mathematics curriculum standards (grade level learning expectations) with vastly different learning expectations that in turn undermines any 'intent' to provide to all students an equal opportunity to learn mathematics. Data from across districts nationally are examined.

### **Do Learning Opportunities Vary by Local District?**

Studies have shown that learning opportunities vary by state (Reys, 2006; Schmidt et al., 2009). Only a few states have mandatory standards for all districts within that state. Since NCLB, many districts have chosen to adopt their state's standards, which is actually their own interpretation of the state standards, frequently tied directly to the textbook series used in the district. One of the primary objectives of the PROM/SE project has been to level the playing field for mathematics and science education within our participating districts. Our goal has been to improve student learning for *all* students and to ensure that *all* students receive an equal, high quality

mathematics and science education. The ultimate questions of intention then may likely be: What do districts intend to cover in mathematics? Do these intentions vary among districts that cross state lines? Do these intentions vary among districts within the same state? Do variations in intended curriculum among districts (across states and within states) matter?

There are over 15,000 local school districts in the United States. Some are quite small and others are very large in geographical area. Some encompass a single small suburb that surrounds a large US city. Others, such as Hawaii, encompass an entire state.

In this report, we have extended our analysis beyond the 61 districts that began with PROM/SE at its inception. Since state standards are not mandatory, and are subject to either interpretation or modification by their school districts, we examined intended topic coverage data for a sample of 101 districts. These districts do not represent a random sample of the entire US. However, they are sufficiently diverse with respect to geographic location, size, achievement levels, and demographics to be considered a microcosm of the entire set of districts across the US.

The bulk of the districts are in Michigan, Ohio, and California. Other districts are in Illinois, Washington, Delaware, New Jersey, Colorado, Pennsylvania, and New York. The districts include US cities such as Chicago, Miami, Cincinnati, Cleveland, Seattle, Rochester, Lansing, and San Diego. The 61 PROM/SE districts from Michigan and Ohio have demographic and assessment performance levels that are characteristic of the US as a whole.<sup>1</sup>

### **How Many Topics Are Intended?**

The intended mathematics curriculum for first through eighth grades is examined using a maximum of 44 topics or content areas. Curricular data collected from all districts indicated that the number of mathematics topics intended for coverage at any grade between first and eighth ranged from 18 to the maximum 44 topics. If the one district with the extremely low value of 18 intended topics is eliminated, the range for the remaining 100 districts was between 26 and 44 topics intended for coverage.

Over 50 districts intended to cover 36 or more topics, up to 44 topics. This means that children in the district that intended to cover only 18 topics would receive exposure to only half or less than half the number of topics as the children in over half the districts. Three-fourths of the districts varied by 10 intended topics or

fewer. To the extent that topics not intended for coverage in some districts are crucial to building a foundation for mathematical skills and literacy, it does not matter whether the number of topics not intended for coverage is a small quantity, between three and five for example. The clear implication is that all districts in the sample did not intend to cover the same content in first through eighth grades.

What topics are most often excluded? Virtually all districts intended to cover in some of grades one through eight topics that represent the standard arithmetic content: addition, subtraction, multiplication, division, fractions, decimals, and percents. However, other topics were frequently excluded in the intended coverage topic list: properties of fractions and decimals; geometric constructions; and slope. For example, only 60 percent of the districts included geometric constructions in their intended topic list. These omissions from the intended coverage topic list are not always significant, as the significance is dependent on the importance of the topics. The most critical topics in the elementary grades are those that build a solid foundation for content taught in middle school and high school.

## At What Grade Levels are Topics Intended?

What topics were intended to be covered in the sample districts? In what grades were those topics intended for coverage? How much did this vary among the districts?

If one considers that for each of eight grades there are 44 possible topics in our analysis framework, then there are 352 (8 x 44) possible topic-grade level combinations, or cells. It is not at all desirable that a district cover each of the topics at each grade level but each cell represents an opportunity for coverage. Figure 1 is a box plot that illustrates the variability among the districts for the 352 topic-grade combinations. Learning opportunities are not equal whenever fewer than all 101 districts in our sample either intended or did not intend coverage of a particular topic at any given grade. There are zero topic-grade combinations that all districts intended to cover and five that none of the districts intended to

cover. Most of the topic-grade combinations were intended to be covered by between 10 and 80 percent of the districts. Of course the largest variation possible would be the case where 50 percent of the districts intended to cover a topic at a given grade and 50 percent of the districts did not. And disappointingly, the typical (median) value for all topic-grade combinations fell at 50 percent. A larger median value would have indicated greater agreement among districts. This implies that across districts children are not only offered severely unequal learning opportunities, but also that we have a huge distance to go to achieve equal learning opportunities for all students. Thus we may conclude that the education a child receives is a function of where the child lives. This finding is similar to that found in studies of state level data. This is only one measure indicating unequal opportunities to learn across districts. Next we explore concepts of coherence and focus to further illustrate the impact of this inequity.

### How to read a Box and Whiskers Plot

A box and whiskers plot, sometimes called a box plot, provides a visual summary of many important aspects of a distribution. The "box" stretches from the 25<sup>th</sup> percentile to the 75<sup>th</sup> percentile, thus containing the middle half of the scores in the distribution. The Median, or 50<sup>th</sup> percentile, is shown as a line across the "box". The "whiskers" stretch from the 25<sup>th</sup> and 75<sup>th</sup> percentiles to the 5<sup>th</sup> or 95<sup>th</sup> percentiles, respectively.

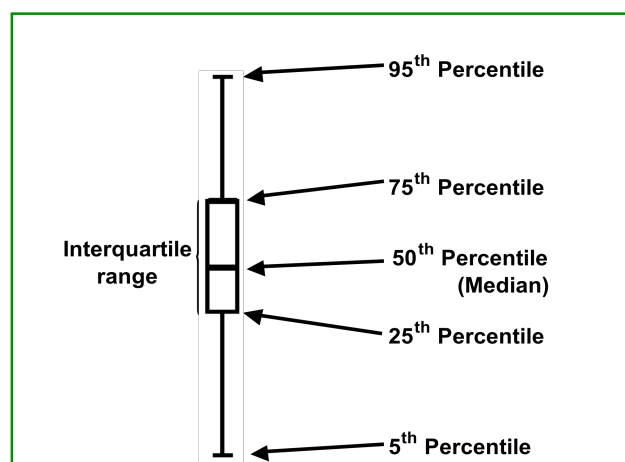
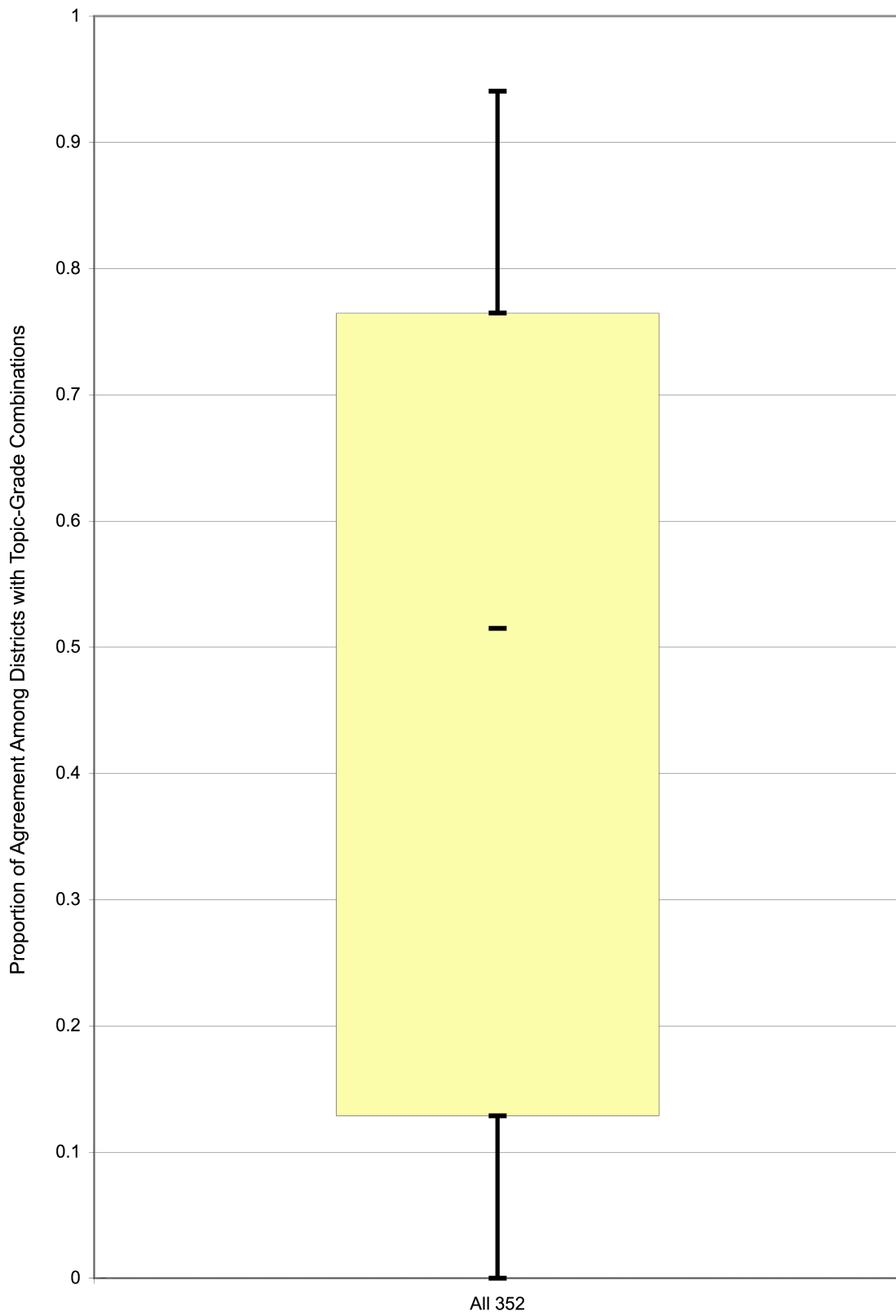


Figure 1. Variability Among Districts in 352 Topic-Grade Combinations



## What is a Coherent and Focused Curriculum?

What are the characteristics of a coherent and focused curriculum? Can these characteristics be identified or measured? We believe that a coherent curriculum introduces and develops topics in a logical sequence. Different topics 'fit' together as part of an integrated, systematic whole, both within a grade level and from grade to grade. Simple concepts are first introduced within simple topics. Topics are developed fully by gradually moving to more complex concepts. Once a topic has been fully developed, it is excluded from the curriculum and other, more complex topics are introduced. A focused curriculum is one that intends a carefully selected and relatively small number of topics, especially in the early grades. The idea is that less is more, in that if fewer topics are included in the curriculum, the few can be addressed in greater depth. The concepts related to them can be developed completely so that students fully understand them. Such an approach facilitates the process of building a strong foundation in mathematics while advancing on to new and more complex topics in succeeding years of study.

One model of a coherent curriculum is depicted in Table 1. It depicts a composite of mathematics curricula intended for grades one through eight of the top

achieving countries (TAC) according to results from the Third International Mathematics and Science Study (TIMSS), completed in 1995. Thirty-two out of the 44 topics from the TIMSS framework that were considered in the previous section are listed in the left column. There are 99 shaded cells that identify the grades in which topics are included in the mathematics curricula in more than half of the TAC (four out of six countries). Thus the shaded cells, representing topic-grade combinations, can be referred to as "coherence cells". The display lists topics in somewhat the same sequence suggested by results from the TAC curricular studies. The sequence of the major topics can be thought of as in a hierarchical structure that concurrently establishes a logical sequence for introducing these topics across the grades.

In addressing the question of whether or not a curriculum is coherent and focused, Schmidt and Houang "developed statistical indicators of both concepts" (Schmidt & Houang, 2007). The matrix in Table 1 is the foundation for their analysis. Columns remain the first eight grades and rows remain the 32 topics, with the sequence unchanged from Table 1. Our matrix has 256 cells (8 x 32). By overlaying the curriculum intended by each of our 101 districts over this silhouetted region, our model of a coherent and focused

curriculum, we can 'measure' the extent of agreement with our model. For any district the 256 cells in the matrix can be divided into three groups:

1) Cells that match the shaded area, displaying agreement with the ideal scenario of coherence as defined by our model – a count of these matches is an indicator of coherence.

2) Cells that are located in the grid in grades before those defined by the shaded region – these cells indicate topics that are covered earlier than that suggested by the ideal scenario of our model.

3) Cells that are located in the grid in grades after those defined by the shaded region – these cells indicate that topics are introduced or covered beyond the time that is recommended by our model.

The sum of the three groups defines the focus. It is a cumulative index and as a result the matrix must be partitioned for each separate grade.<sup>2</sup> Though these measures are not developed with these data in this article, other measures are developed in this article to examine coherence and focus.

Figures 2 and 3 display coherence and focus data, respectively, that depicts variation among our sample districts. Again marked differences in learning

opportunities are indicated. Notably, there were no "coherence cells" for which all districts intended coverage or for which no districts intended coverage (Figure 2, the plot labeled 'UTT 99'). As described earlier, with provision of a coherent curriculum together with equal learning opportunities across all sample districts as our ideal situation, we would expect a perfect fit with the coherence model used in this analysis. A perfect fit would be indicated by a value of 100 percent in each cell. This was far from the case.

The typical (median) percentage of districts that intended to cover one of the topic-grade combinations in agreement with the coherence model (where 99 key topic-grade combinations are designated for intended coverage) was 83 percent. Between 73 and 89 percent of the sample districts included half of these 99 key topic-grade combinations in their intended curriculum. The district data were slightly more variable than were the state data examined in another study. And the variance of the district data from the defined model of coherence was slightly greater than what was portrayed in the study that examined state data. All of this taken together suggests the possibility of unequal learning opportunities and vast differences in the potential for delivering a coherent mathematics program based on intended curriculum among school districts.

Table 1. Mathematics Topics Intended at Each Grade by a Majority of TIMSS 1995 Top-Achieving Countries

Topic	Grade							
	1	2	3	4	5	6	7	8
Whole Number: Meaning	●	●	●	●	●			
Whole Number: Operations	●	●	●	●	●			
Measurement Units	●	●	●	●	●	●	●	
Common Fractions			●	●	●	●		
Equations & Formulas			●	●	●	●	●	●
Data Representation & Analysis			●	●	●	●		●
2-D Geometry: Basics			●	●	●	●	●	●
2-D Geometry: Polygons & Circles				●	●	●	●	●
Measurement: Perimeter, Area & Volume				●	●	●	●	●
Rounding & Significant Figures				●	●			
Estimating Computations				●	●	●		
Whole Numbers: Properties of Operations				●	●			
Estimating Quantity & Size				●	●			
Decimal Fractions				●	●	●		
Relation of Common & Decimal Fractions				●	●	●		
Properties of Common & Decimal Fractions					●	●		
Percentages					●	●		
Proportionality Concepts					●	●	●	●
Proportionality Problems					●	●	●	●
2-D Geometry: Coordinate Geometry					●	●	●	●
Geometry: Transformations						●	●	●
Negative Numbers, Integers, & Their Properties						●	●	
Number Theory							●	●
Exponents, Roots & Radicals							●	●
Exponents & Orders of Magnitude							●	●
Measurement: Estimation & Errors							●	
Constructions Using Straightedge & Compass							●	●
3-D Geometry							●	●
Geometry: Congruence & Similarity								●
Rational Numbers & Their Properties								●
Patterns, Relations & Functions								●
Proportionality: Slope & Trigonometry								●
Number of Topics Intended	3	3	7	15	20	17	16	18
Intended by more than half of the top-achieving countries					●			



Figure 2. Variability Among Districts in 256 Topic-Grade Combinations and the 99 Upper Triangle Coherence Cells (32 Topics)

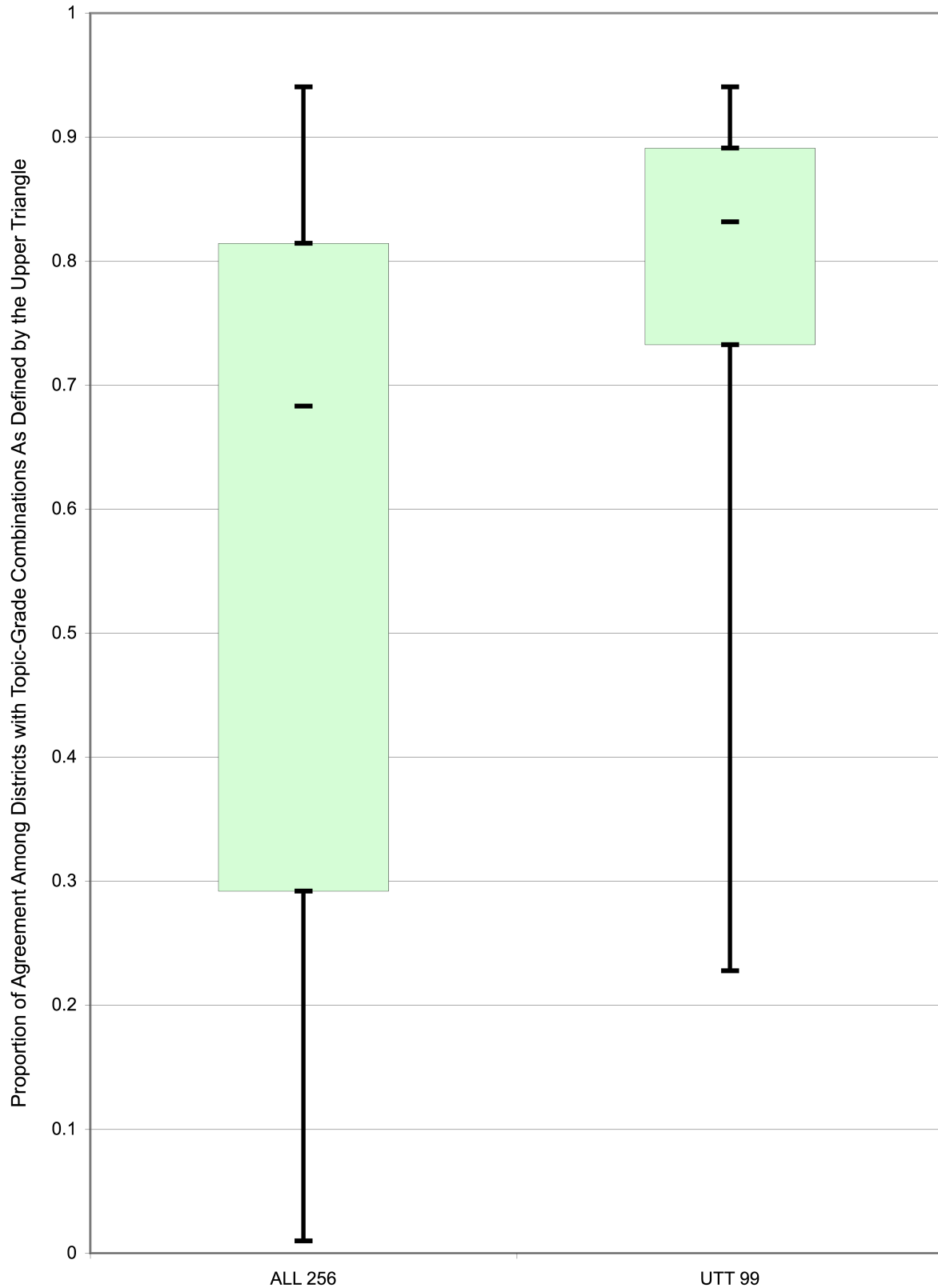
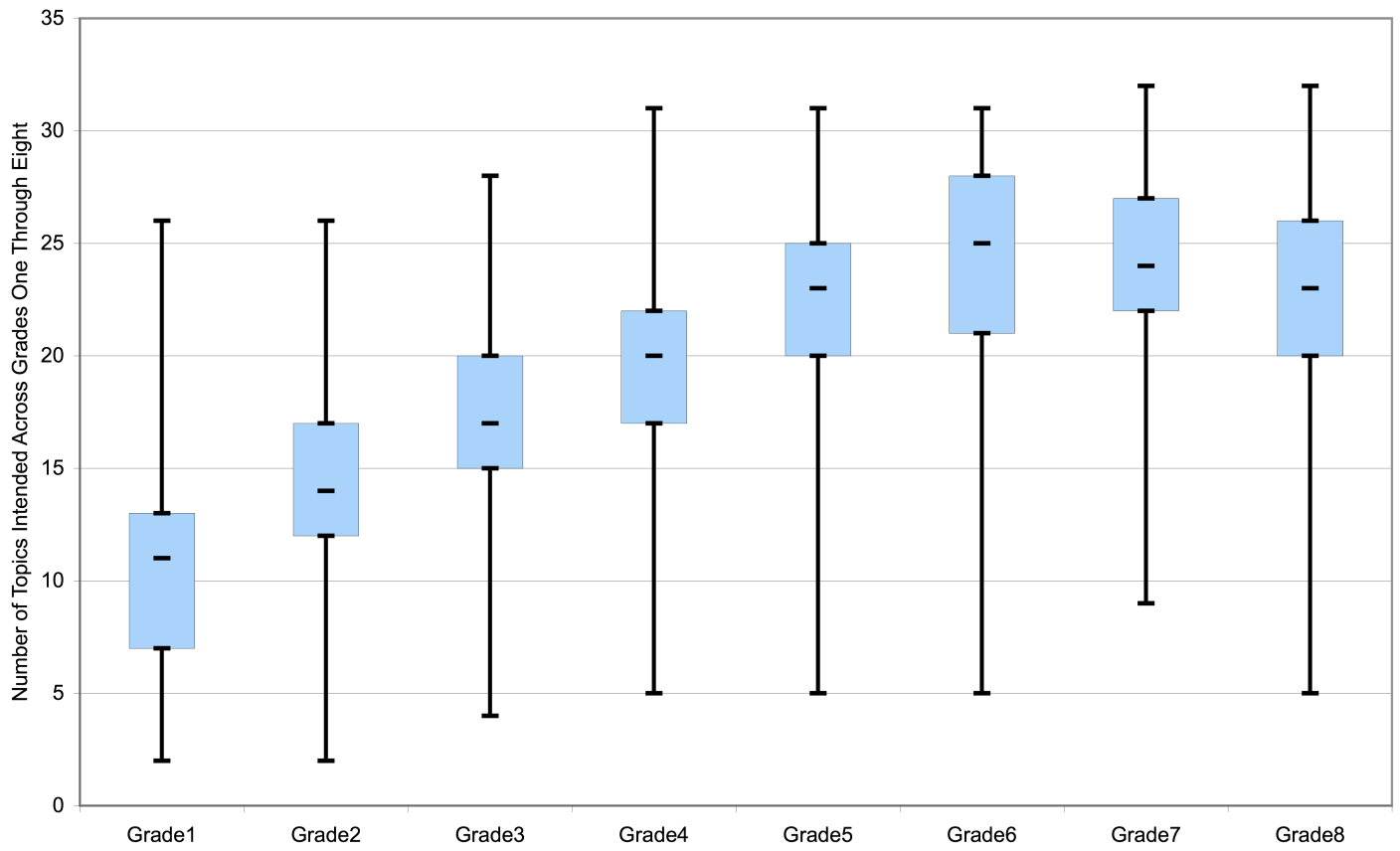


Figure 3. Variability in Number of Topics Intended Across 100 Sampled Districts



A sense for the amount of focus in the sample districts' curricula can be gained by examining the variability data in Figure 3. The data indicated substantial variability across districts both in the number of topics intended at each grade as well as over the first eight years of schooling. As noted earlier, topics in a focused curriculum will be fewer in number than for a curriculum that is not focused. This difference is evident if we compare the number of topics intended across grades one through eight from the TAC coherence model – (3, 3, 7, 15, 20, 17, 16,18) with the number of topics that are intended in Figure 3 at the median (11, 14, 17, 20, 23, 25, 24, 23). Even if we consider the number of topics that are intended across the grades at the 25<sup>th</sup> percentile of Figure 3 (7, 12, 15, 16.75, 20, 21, 22, 20), the number of intended topics is greater than intended in the coherence model for all grades except grade 6. Another way of thinking about this is that more than 75 percent of the districts intended to cover more topics per grade level than were indicated by the TAC countries' profile. Clearly most of the district curricula are not focused.

In spite of this diffused curricula, important topics were often not included. The greatest variability is evident with key topics that build a foundation for understanding basic number concepts. These topics include:

properties of whole number operations; properties and relationships of fractions and decimals; prime numbers; and exponents/orders of magnitude. Only 68 percent of the districts on average intended to cover them during the critical grades for covering them (according to the coherence model). One-third of the districts do not include these topics during the critical grades. This is one more important difference in potential learning opportunities and another indication of inequities in educational experience.

There was also substantial variation among districts in content areas related to geometry, especially key content areas including geometric relationships such as coherence and symmetry and the two-dimensional Cartesian coordinate system. On average the content coverage intentions of only 69 percent of the sample districts matched the topic-grade combination cells in the coherence model. Again it appears that students in close to one-third of the districts will not have the opportunity for exposure to these topics at the most appropriate grades for learning this content. These unequal learning opportunities are, considered alone, significant. Their omission from the intended curriculum during critical grades negatively impacts the development of a deeper understanding of mathematics.

Understanding of these topics takes students beyond simple computations to a point where they are able to reason quantitatively and use spatial skills that are much more in demand in today's global economy.

Coherence and focus are two important concepts that studies have linked to student achievement levels. Data that characterize these concepts portrayed substantial variability among districts. Together with whether or not topics are intended for coverage, the sequence in which topics are introduced and developed also makes a difference in learning opportunities. Topics introduced according to a sequence that adheres to the logical structure of mathematics are more likely to be understood by students. What is more meaningful to them is more likely to be remembered. Variability in topic sequencing substantially affects students' ability to understand content. Variability in topic sequencing is the most important source of unequal learning opportunities.

### **How Much Do Districts Within a State Vary?**

Why consider the extent to which the intended curricula for districts within a state vary? We have just demonstrated that there is a large amount of variation across our 101 districts, a sample drawn from 11 states. Isn't this enough? Would we find an equally large variation if we

examined only the districts within a state, or would their collection of intended topics for first through eighth grades be more homogeneous? We need to explore this question to determine whether equal learning opportunities exist within a state. This is worth examining to further support our claim that it matters where a child lives when taking into account equal learning opportunities.

We will examine more closely the data for three states from which we have sampled a sizable number of districts: Michigan (28 districts); Ohio (33 districts); and California (25 districts). The districts from Michigan and Ohio do not necessarily constitute a representative sample across the entire state. The California districts were selected in order to represent the entire state. The results can be most representative for California, only slightly less so for Ohio and then Michigan. By implication, results may also be generalizable to the other states.<sup>3</sup>

With the advent of NCLB legislation there has been a greater push on the part of the states to influence the curriculum at the district level, especially for the subjects targeted by NCLB – mathematics, science, and language arts/reading. Results of state tests are publicly reported locally and statewide. Results from state assessments, which are based on state

standards, are used to determine whether or not districts are progressing in accordance with NCLB goals. The question we explore here is: Are the districts within a state aligning to their state standards, thus becoming more homogeneous in the topics that they intend to cover than districts from outside their state?<sup>4</sup>

Figure 4 shows the distribution of the percentage of districts within each of the three states examined. The distribution of their coverage of the 99 topic-grade combinations as defined by the coherence model is depicted. Most districts in Michigan and Ohio intended to cover most of the topic-grade combinations in the coherence model. The percent distributions matching the coherence model in Michigan ranged from around 35 to 95 percent of the districts. The typical (median) value was around 80 percent. The distribution in Ohio was quite different. The range of distributions was from 3 to 97 percent. The typical (median) value was 97 percent. This suggests that for a typical topic-grade combination among the 99 coherence cells, 97 percent of the districts intended coverage. In Michigan the middle half of the 99 topic-grade coherence cells (the data points that fall between the 25<sup>th</sup> and the 75<sup>th</sup> percentiles) ranged between 71 and 93 percent of the districts. In Ohio the corresponding range was between 94

and 97 percent of the districts. As indicated by the medians, the variability for these topic-grade combinations that define a coherent curriculum was relatively small. Ohio districts were more homogeneous than Michigan districts.

In spite of this pattern, both states have topic-grade combinations in coherence cells in which there was greater variability. For example, in Michigan only 43 percent of the sample districts intended coverage of proportionality concepts in grade five. In the Ohio sample only 15 percent of the districts intended to include a related topic, proportionality problems, in grade five. This variability may seem trivial, but these topics are used as examples because developing a deep understanding about them is a significant part of building a strong foundation for more complex content areas that will be introduced in the middle grades and further developed in algebra and other high school courses.

The data discussed to this point in this section appear to support the suggestion that variability among districts within states is less than across districts from many states. Yet there is another part of this story. It has to do with focus in the intended curriculum. How much variability is there across the 157 (256 – 99) topic-grade cells that do not fall into the shaded

area that is our model for a coherent curriculum? These are the combinations that are not typically covered by the top-achieving countries. Or another way of saying this is that coverage of these topics is in contradiction to the concept of a coherent curriculum. This also suggests that these topics are covered at grades before prerequisite topics have been properly covered, or are intended in the curriculum for longer than necessary so that teachers have difficulty devoting time to other topics that need more development at the grade level in question.

Figure 5 illustrates that the variability was much greater in these cells. The result is likely a negative impact on the intended coverage of the areas that matched with the defined coherence model. Coverage of areas outside the coherence model decreases the focus, as it reduces the amount of available instructional time for content defined by the coherence cells. Including instruction in these areas may also tend to confuse students about the logic of the mathematics content being developed.

The variability in both Michigan and Ohio data for the topic-grade combinations that fall outside the coherent curriculum model was substantial. Values ranged from zero to 97 percent of the districts. The typical

(median) value was 46 percent in Michigan and 35 percent in Ohio, with around 39 percent as median value for data from both states. These differences suggest a large variability which is comparable to that seen when data are examined from the 101 sample districts from 11 states.

The conclusion about variability across within state districts is thus more complex than it first appeared. In one respect districts within the same state are more homogeneous than all districts with respect to intended coverage of the topic-grade combination cells as defined by our coherence model. However within states, districts are as diverse as the entire sample when data for topic-grade combinations that do not fit our coherence model are examined.

There is yet another way to examine this issue of variance among districts. We might turn our attention to the subset of topics referred to earlier – topics that American students must fully understand if they are to compete internationally. These topics are taught in middle school. Deep understandings of these topics contribute to the transition from elementary arithmetic to more formal mathematics – the kind of mathematics necessary in our increasingly technological economy. Another question arises with

this scenario. How much does intended coverage vary within state districts for these key transitional topics?

In Ohio almost all districts intended to cover most key topics at the grades designated in the defined coherence model. The data for two of the topics do not fit this pattern. Three-dimensional geometry was intended to be covered in eighth grade by only 44 percent of the districts. A topic related to important number concepts, primes and factorization, was intended to be covered by only 40 percent of the Ohio districts. These data generally support the implication that the Ohio districts are more homogeneous with respect to topics they intend to cover. However for certain critical topics their intentions are quite variable and not consistent with a coherent curriculum as defined by our model.

California's case is quite different. The data depicted in Figure 4 show a large variation among the California districts used in our sample. The percent of districts that intended to cover each of the topic-grade combinations defined by our coherence model ranged from 19 to 96 percent. The typical (median) value was 74 percent. These results were much more variable than was the case for the

districts within Michigan and Ohio. More than 55 percent of the districts in California intended to cover most of the topic-grade combinations that are identified in our coherence model.

The California data are older, collected in 1997 and 1998 as California was still struggling with the transition to a new and more challenging framework, the 1992 California Mathematics Framework. The new framework caused a major controversy in California, one that peaked in 1997 and has been dubbed the "Math Wars." Over a decade later the controversy is still ongoing.<sup>5</sup> Per California state law a 30 month interval is required between establishment of new criteria and adoption of it. Therefore after the 1992 framework was adopted, related materials were not purchased and the new curriculum was not implemented until the 1995-1996 school year. The greater variability for the California districts is likely a consequence of the short time frame between adoption and data collection as well as the uncertainty and instability that was still roiling as California districts struggled with the mathematics education reform effort initiated by the state.

Figure 4. Variability Among Districts Within Selected States in Coverage of 99 Coherence Cells

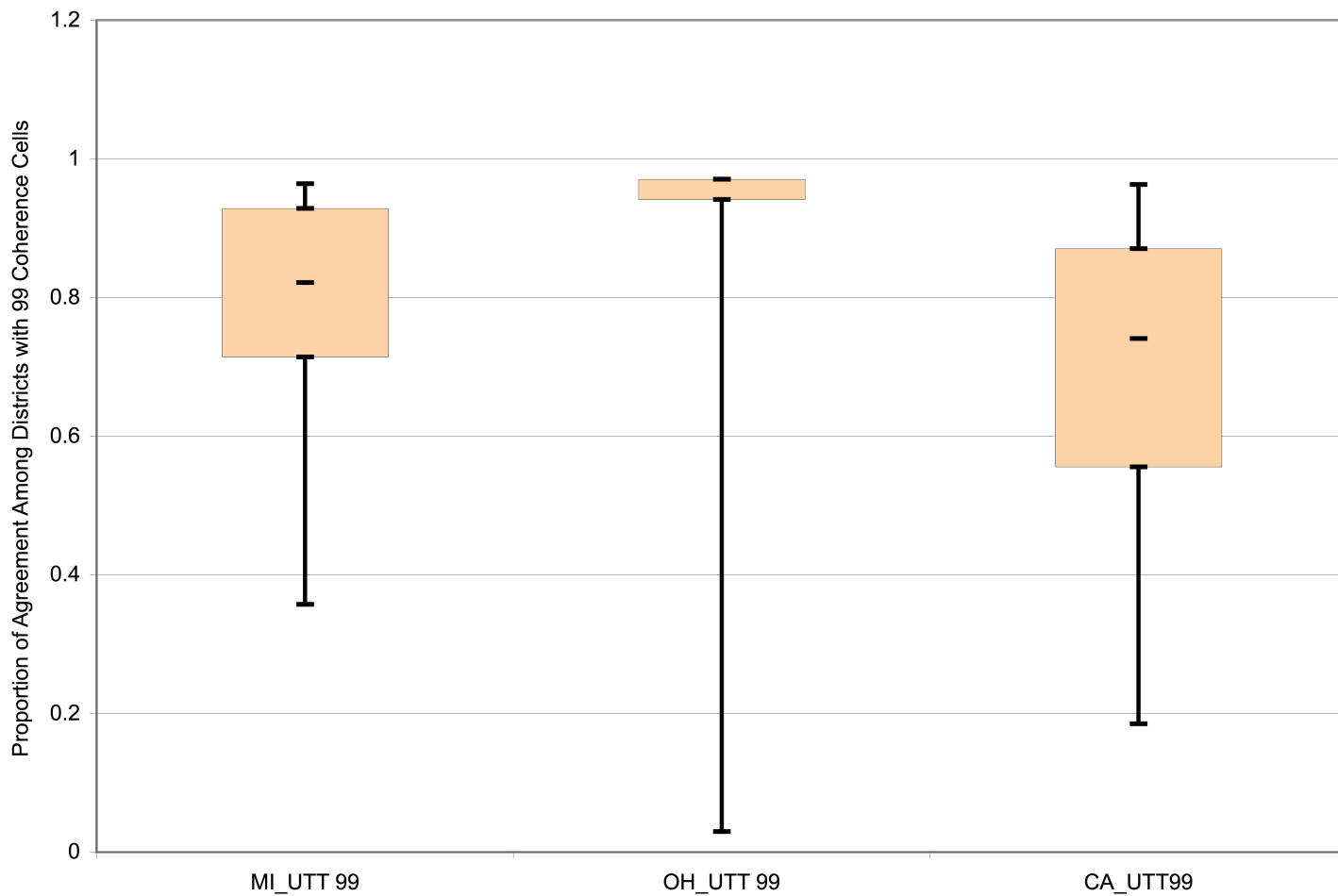
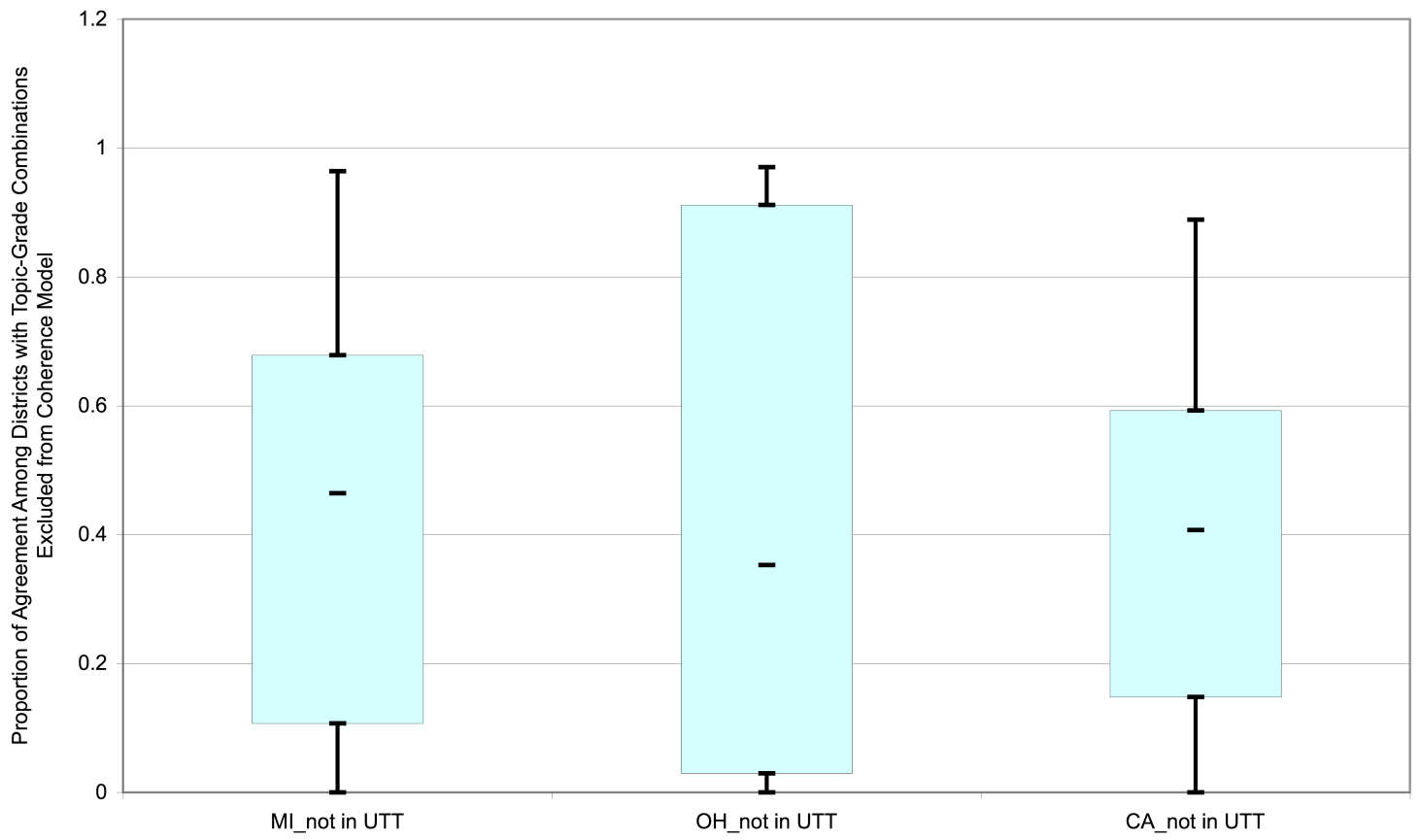




Figure 5. Variability Among Districts Within Selected States in Coverage of Topics That Are Excluded from Coherence Model



How much variability exists for the topic-grade combinations outside the region that defines coherence in our model? The percentage of California districts that intended to cover the content represented by cells that lie outside this coherence region was extremely variable. The results were similar to those just presented for Ohio and Michigan. The typical (median) value for California districts was 41 percent. Another way of thinking about this is that for most of the cells that are outside the coherence region three out of five California districts in the sample did not intend to cover this content, but two out of five California districts did. The implications for lack of focus and less clarity (less coherence) in the mathematics curriculum are just as strong across California districts as they appear to be for Michigan and Ohio.

Across the three states examined we therefore see sizeable variation in opportunities to learn among districts within each state. Of the three, Ohio appeared to be more homogeneous when only the topic-grade combination cells that correspond with our coherence model are considered. But when topic-grade cells outside the coherence region are taken into account, districts within all three states showed considerable variability. Our concern here is that content is being covered out of a preferred sequence,

either too early or too late, as defined by our coherence model. Students may not have acquired the prerequisite knowledge necessary to fully understand content covered in grades earlier than specified by the model. And if content is intended for coverage in grades beyond the time indicated in the coherence model, there is less instructional time for other mathematics content. If one adds to this concerns about whether or not key topics are intended at specific grades in a coherent fashion, all states intended to cover some of the key topics according to the coherence model. But, here too, there is variation among the three states.

All of this suggests that adopting state standards may facilitate some reduction in variation in intended coverage across its districts but not substantially. This is particularly true for the topic-grade cells that lie outside the region defined by our coherence model. The implication is that while lacking coherence and focus, mathematics' logical structure will not be developed and learning opportunities across districts will be inequitable.

### **A Special Look at Algebra and Geometry**

For most of the world, especially those that are our economic competitors, the middle school curriculum focuses on content areas related to algebra and

geometry. However these topics do not suddenly appear in eighth grade. Fifth and especially sixth and seventh grades are transition years. Very basic concepts that will build a foundation for algebra and geometry in grade eight and beyond are developed in these years.

What is the variability across districts for these critical topic areas? To explore this we return to our consideration of all 101 districts in our sample. Most of the topic-grade combinations that define algebra content were intended to be covered by over 90 percent of the districts. One topic that tends not to be covered is coordinate geometry. Does this omission matter? Though this may be considered a geometry topic it is one that is critical to the development of essential concepts related to algebra. Equations of lines, an important algebraic concept, may be represented using coordinate geometry. The inclusion of coordinate geometry may facilitate the understanding of slope in linear functions. Clearly it is a concept central to algebra that should be a part of every mathematics program.

According to our model for coherence, coordinate geometry should be introduced in the fifth grade, further developed in grades six and seven, and used in the eighth grade as it is linked to algebra. The data indicate that only around 40 percent

of the sampled districts intended coverage of this topic during grades six and seven. Almost all districts introduced it in the fifth grade, and intended to cover it again in eighth grade. The gap in continuous intended coverage of this topic likely affects coherence in more than half the districts that did not intend to cover this topic in sixth and seventh grades.

Moving the discussion beyond algebra to geometry, a topic typically not well covered in the American educational system, TIMSS data indicated that this area is among the weakest in the US in terms of student performance. Between 80 and 85 percent of the sampled districts intended to cover most of the seven topics that define the geometry curriculum in our framework. Given the central position of geometry to mathematics, this is less than desirable. It means that students in one out of five districts that we sampled are unlikely to be taught this critical content during their first eight years of schooling.

As described above, even fewer districts intended to cover the topic coordinate geometry. Though less central to the development of mathematical knowledge, two additional topics – geometric transformations and constructions – were intended by a low percentage of districts. Geometric constructions is the topic least intended; only 30 percent of the sampled

districts intended to cover this topic. This indicates variability in learning opportunities among the sampled US districts, but it also indicates deviation from what students in the high achieving countries are being taught.

## Conclusions

This paper explored at the most general levels of the US educational system whether there is equity across districts in learning opportunities. Are the American schools, as the rhetoric suggests, great equalizers? Do all children receive the same educational opportunities upon which to build their futures and achieve the great American dream? At least at the state and district level, the answer is a simple “no”. Not at least in mathematics, but there is little reason to believe it is different for other subjects.

Location matters. The states and districts within a state, through their published curricular intentions, impact the topics to which a student will be exposed. State standards are vastly different and impact learning opportunities nationwide. The variability that exists at the district level is even more disturbing. Districts can be separated by only a couple of miles and so the phrase “born on the wrong side of the tracks” seems appropriate but may be recast as “born on the wrong side of the

street”, at least the street demarcating district boundaries.

It is difficult to imagine what it is like to live a very different life than the one we do. So although in principle we may be opposed to having different learning opportunities for children living in very different socioeconomic conditions, it might seem reasonable as to why that might be the case. While lamenting the situation that the unequal learning opportunities exist, it might just seem like the “way it is” with little that is possible to be done to correct it.

It is not acceptable, however, to settle for the “way it is” with the thought that this applies to some and not to others. The data in this paper bring this issue to the front door step of every American home. The data show that such disparities in learning opportunities are also about their children and others like them who live in nearly identical socioeconomic conditions. Their opportunities to learn are being influenced by such ordinary things as where their parents just happen to live – which state, and within the state in which one of the two wonderful adjacent communities, that seem so identical to each other in every aspect, but one.

Imagine this scenario, based on an actual experience. A family was living in a home

that is just about three miles outside a city proper with a population of just over 40,000. The city is home to a public university that carries an enrollment of almost 21,000 students. The city has its own school system, which is separate from the school system that serves the remainder of the county. The standards between the two school systems are quite different. Because of geographical boundaries the children were unable to attend the city school even though one of the parents worked at the university in the city a few miles away from their home. The result was that the children were exposed to an entirely different set of learning opportunities. Most American parents are just not aware of such inequities – that a few miles of difference in the location of one’s home could make a big difference in what learning opportunities their children will have.

This is of course true from state to state as well. And whereas some families study intensively the community in which they will reside, with an eye toward the school district that tends to have the best results, families are often powerless to choose the state in which they will reside. The good news is that educational opportunities are now one consideration of some businesses that are looking to relocate. But much of the American public remains less aware of these issues.

The simple point of this paper is that these disparities in learning opportunities exist. Given the cumulative nature of knowledge, especially in mathematics, differences in learning opportunities lost at a specific grade may not be gained at a later time. In fact, the opposite is more likely the case especially but not limited to the 70 to 80 percent who never graduate from college. These disparities are not just experienced by children who live in poverty. This affects children who live in wealthy suburbs that surround urban areas as well. We must strive for more equity in learning opportunities so that **all** students do in fact receive a high quality education.

## Endnotes:

<sup>1</sup> In Michigan and Ohio using the TIMSS national achievement results as a base we compared those results with the results from the testing of children in grades three, four, seven and eight in the PROM/SE project. The results were not statistically significantly different. We also compared the districts in terms of demographics and similarly found no significant differences from national estimates.

<sup>2</sup> A more detailed description about this can be found in Schmidt, W. H. and Houang, R. T. (2007). Lack of Focus in the Mathematics Curriculum: Symptom or Cause? (Chapter 4). In: T. Loveless (Ed.), *Lessons learned, What international assessments tell us about math achievement*. Washington, DC: Brookings Institution Press.

<sup>3</sup> For Ohio, though the sample is not random, it represents some 13 percent of the state's student population, and with both Cleveland and Cincinnati included is a reasonably representative sample. Michigan's sample does not include the second and third largest cities as does Ohio and is less representative, especially of the urban population. The California districts were sampled with probabilities proportional to the size of the district as indicated by the total student enrollment.

<sup>4</sup> At one level this is the question of alignment that has become a big policy issue and has been explored by Porter and others, especially the alignment between state standards and tests. See Porter (2004).

<sup>5</sup> See Becker & Jacob (1998), Saunders (2008) and Sowder (1998).





Promoting Rigorous Outcomes in Mathematics and Science Education (PROM/SE) is a comprehensive research and development effort to improve mathematics and science teaching and learning in grades K-12, based on assessment of students and teachers, improvement of standards and frameworks, and capacity-building with teachers and administrators.

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